Solving Sinusoidal Equations Algebraically

Example Given $y = 20\sin(3(x-1))^\circ + 50$, solve for x when $y = 60$.

* sub in 60 for y: ← need to isolate. just like: $60 = 20t + 50$

$$60 = 20\sin(3(x-1))^\circ + 50$$

$$60 - 50 = 20\sin(3(x-1))^\circ$$

$$\frac{10}{20} = \frac{20}{20}\sin(3(x-1))^\circ$$

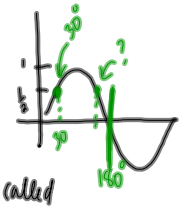
$$\frac{1}{2} = \sin(3(x-1))^\circ$$

let $\theta = 3(x-1)$

$$\frac{1}{2} = \sin\theta \quad \text{solve for } \theta$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 30^\circ$$



Your calculator only gives you one solution... this is called the primary angle. If the function is sine then the secondary angle can be found:

$$\theta_1 + \theta_2 = 180^\circ \quad \text{for sine}$$

$$\theta_1 + \theta_2 = 360^\circ \quad \text{for cosine}$$

So now we have $\theta_1 = 30^\circ$ and $\theta_2 = 150^\circ$

Recall: $\theta = 3(x-1)$

$$\frac{30^\circ}{3} = \frac{3(x-1)}{3} \quad \text{and} \quad \frac{150^\circ}{3} = \frac{3(x-1)}{3}$$

$$10 = x - 1 \qquad 50^\circ = x - 1$$

$$\boxed{x = 11^\circ} \qquad \boxed{x = 51^\circ}$$

We also need to know the period: ← from your eq:

$$3 = \frac{360}{\text{period}}$$

$$\text{period} = \frac{360}{3}$$

$$\boxed{\text{period} = 120^\circ}$$

$$x = \left\{ \begin{matrix} 11^\circ + 120^\circ k \\ 51^\circ + 120^\circ k \end{matrix} \right\} k \in \mathbb{Z}$$

(from θ_1; from θ_2; 120° ← period)

Example

A rabbit population can be modeled by the equation:

$$P = 50\cos(30(t-2002))^\circ + 350$$

(Amp, $\frac{360}{P}$, PS, SA)

where t is in years.

a) Determine the population in 2010 (i.e. $t = 2010$)

$$P = 50\cos(30(2010-2002))^\circ + 350$$
$$P = 50\cos(30(8))^\circ + 350$$
$$P = 50\cos(240)^\circ + 350$$
$$\boxed{P = 325}$$

There were 325 rabbits in 2010.

b) Determine when (i.e. in what year) the rabbit population reaches 375

$$375 = 50\cos(30(t-2002))^\circ + 350$$
$$375 - 350 = 50\cos(30(t-2002))^\circ \qquad 375 = 50\% + 350$$
$$\frac{25}{50} = \frac{50}{50}\cos(30(t-2002))^\circ$$
$$\frac{1}{2} = \cos(30(t-2002))^\circ \quad \theta$$

Let $\theta = 30(t-2002)$

$$\frac{1}{2} = \cos\theta$$
$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$
$$\theta = 60^\circ \leftarrow \text{primary angle } (\theta_1)$$

For cosine: $\theta_1 + \theta_2 = 360^\circ$

$$60^\circ + \theta_2 = 360^\circ$$
$$\theta_2 = 300^\circ \leftarrow \text{secondary angle } (\theta_2)$$

Recall: $\theta = 30(t-2002)$

So: $\frac{60^\circ}{30^\circ} = \frac{30(t-2002)^\circ}{30^\circ}$ and $\frac{300^\circ}{30^\circ} = \frac{30(t-2002)^\circ}{30^\circ}$

$2 = t - 2002$ (starting points) $10 = t - 2002$

$\boxed{t = 2004}$ $\boxed{t = 2012}$

We still need to know the period:

$$30 = \frac{360}{\text{period}}$$
$$\text{period} = \frac{360}{30}$$
$$\boxed{\text{period} = 12}$$

$$x = \{2004 + 12k \mid 2012 + 12k\}, k \in \mathbb{Z}$$